MIRAGE: Elements on the structure of the model

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1 Notations

The i and j indices refer to sectors, r and s refer to regions, t to periods.
Superscripts for prices P refer to the related variable.
$U(s)$ is the subset of countries in the same development level as region s and
$V(s)$ is the subset of countries with a different level of development.
$Agri(i)$ is the subset of sectors from agriculture.
i_{TrT} refers to transport sectors and r_{EU} refers to the European Union regions.
The reference year is indexed with t_0.

2 Parameters definition

σ_{VA_j} σ_{CAP_j} σ_C σ_{IC} σ_{KG} σ_{GEO_i} σ_{ARM_i} σ_{IMP_i} σ_{VAR_i}	Substitution elasticities of factors and goods demand
$cmin_{i,r}$	Minimal consumption of good i in the final demand of region r
epa_r	Saving rate in region r
$\mu_{i,r,s}$	Transport demand per volume of good
θ_r	Value share of region r transport sector in the world production of transport
$DD_{i,r,s,t}$	*Ad-valorem* tariff rate applied by regions s on its imports from region r
$MaxExpSub_{i,r,t}$	Maximum level of subsidized exports authorized by the WTO
$taxp_{i,r}$ $taxcc_{i,s}$ $taxicc_{i,s}$ $taxkgc_{i,s}$	Tax rates applied on production, final consumption, intermediate consumption and capital good
$taxAMF_{i,r,s}$	Export tax rate equivalent to the Multifibre Arrangement
$TsubK_{i,r}$	Subsidy rate on capital
$TsubTE_{i,r}$	Subsidy rate on land
$cf_{j,r}$	Fixed cost per firm, in units of output (imperfectly competitive sectors)
$mmoy_{i,r}$	Mark-up average
$Quota_{i,r,t}$	Maximum production in sectors where quotas hold
α	Elasticity of investment to capital return rate
$\gamma^L_{i,r}$ $\gamma^Q_{i,r}$ $\gamma^{TE}_{i,r}$ $\gamma^{RN}_{i,r}$	Value share of factors in value added (Cobb-Douglas)

δ	Depreciation of capital
$\rho_{r,t}$	Population growth rate of region r (World Bank data)
a_{XXX}	Various share and scale coefficients in CES or Cobb-Douglas functions
$PGF_{r,t}$	Total factor productivity

3 Variables definition

Production	
$Y_{i,r,t}$	Output of sector i firms
$VA_{i,r,t}$	Value added
$CNTER_{i,r,t}$	Aggregate intermediate consumption

Factors	
$Q_{i,r,t}$	Aggregate of human capital and physical capital
$L_{i,r,t}$	Unskilled labour
$L^{Agri_{i,r,t}}$	Total Unskilled labour in agriculture
$L^{notAgri}_{i,r,t}$	Total Unskilled labour in sectors other than agriculture
$TE_{i,r,t}$	Land
$RN_{i,r,t}$	Natural resources
$H_{i,r,t}$	Skilled labour
$K_{i,r,s,t}$	Capital stock from region r to region s in sector i
$KTOT_{i,r,t}$	Total capital stock in sector i and region r
$\overline{L}_{r,t}$	Total supply of unskilled labour
$\overline{TE}_{r,t}$	Total supply of land
$\overline{H}_{r,t}$	Total supply of skilled labour
$\overline{K}_{r,t}$	Total supply of capital

Demand	
$BUDC_{r,t}$	Budget allocated to consumption
$UT_{r,t}$	Utility
$P_{r,t}$	Price of utility
$C_{i,r,t}$	Aggregated consumption
$IC_{i,j,r,t}$	Intermediate consumption of good i used in the production of sector j
$INVTOT_{r,t}$	Total investment in region r
$INV_{i,r,s,t}$	Investment from region r to sector i in region s
$B_{r,t}$	Investment scale coefficient
$KG_{i,r,t}$	Capital good demand of sector i in region r
$DEMTOT_{i,r,t}$	Total demand
$DEMU_{i,r,t}$	Total demand, in region r, of good originating from regions with the same development level as region r (including local demand in region r)
$DEMV_{i,r,t}$	Total demand, in region r, of good originating from regions with a different development level from region r

$D_{i,r,t}$	Domestic demand of good i
$DVAR_{i,r,t}$	Domestic demand of good i produced by each firm of region r
$M_{i,r,t}$	Total demand, in region r, of good i originating from regions with the same development level as region r other than region r
$DEM_{i,r,s,t}$	Demand, in region s, of good i originating from region r
$DEMVAR_{i,r,s,t}$	Demand of good i produced by each firm of region r

Transportation sector

$TRADE_{i,r,s,t}$	Exports to region s of industry i in region r
$TR_{i,r,s,t}$	Transport demand
$MONDTR_t$	Transport aggregate
P_t^T	Transport of commodities price
$TRM_{i,r,t}$	Supply of international transportation sector i in region r

Monopolistic competition

$EP_{i,r,s,t}$	Perceived price elasticity of total demand
$EPD_{i,r,t}$	Perceived price elasticity of domestic demand
$NB_{i,r,t}$	Number of varieties in imperfectly competitive sectors
$SDU_{i,s,t}$	Market share of domestic demand in demand of regions with the same level of development as region r
$SDT_{i,s,t}$	Market share of domestic demand in total demand
$SE_{i,r,s,t}$	Market share of imports from region r in imports of region s originating from regions with the same level of development
$SU_{i,r,s,t}$	Market share of imports from region r in demand of region s for goods from regions with the same level of development
$SV_{i,r,s,t}$	Market share of imports from region r in imports of region s originating from regions with a different level of development
$ST_{i,r,s,t}$	Market share of imports from region r in demand of region s

Tax revenue

$RECPROD_{i,r,t}$	Revenue of production tax
$RECDD_{i,r,t}$	Revenue of tariff
$RECCONS_{i,r,t}$	Revenue of consumption tax
$RECEXP_{i,r,t}$	Revenue of exports tax
$RECTAX_{r,t}$	Total tax revenue
$RQUOTA_{i,r,s,t}$	Implicit transfers due to quotas
$REV_{r,t}$	Regional revenue
$SOLD_{r,t}$	Current account balance
$PIBMVAL_t$	Total GDP in value
$GDPVOL_{r,t}$	Regional GDP

Prices and taxes

P^{XXX}	Generic notation to indicate the price of the variable XXX
$P^{CIF}_{i,r,s,t}$	CIF price
$P^{Int}_{i,t}$	Intervention price (European Union only)
$W^{K}_{r,t}$	Capital return rate in region r
$W^{K}_{i,r,t}$	Capital return paid to the investor
$W^{TE}_{r,t}$	Land return rate in region r
$W^{TE}_{i,r,t}$	Land return rate paid to the owner
$TAXEXP_{i,r,s,t}$	Export tax rate
$TAXREF_{i,r,s,t}$	Auxiliary variable to adjust TAXMOY to its proper level while keeping unchanged the distribution across destinations
$TAXMOY_{i,r,t}$	Average export tax rate across destinations

4 Equations of the model

4.1 Supply

Determination of supply results from the following optimization programs:

Leontieff relation between value added and intermediate consumption:

Imperfect competition

$$\min NB_{i,r,t} P^{Y}_{i,r,t}(Y_{i,r,t} + cf_{i,r}) = P^{VA}_{i,r,t} VA_{i,r,t} + P^{CNTER}_{i,r,t} CNTER_{i,r,t} \quad (1)$$

$$\text{s.t.} \quad NB_{i,r,t}(Y_{i,r,t} + cf_{i,r}) = a^{VA}_{i,r} VA_{i,r,t} = a^{CNTER}_{i,r} CNTER_{i,r,t} \quad (2)$$

Perfect competition

$$\min P^{Y}_{i,r,t} Y_{i,r,t} = P^{VA}_{i,r,t} VA_{i,r,t} + P^{CNTER}_{i,r,t} CNTER_{i,r,t} + P^{Quota}_{i,r,t} Quota_{i,r,t} \quad (3)$$

$$\text{s.t.} \quad Y_{i,r,t} = a^{VA}_{i,r} VA_{i,r,t} = a^{CNTER}_{i,r} CNTER_{i,r,t} \quad (4)$$

For sectors where quotas hold (perfect competition only):

$$Y_{i,r,t} = Quota_{i,r,t} \quad (5)$$

Factor demand

$$\min P^{VA}_{i,r,t} VA_{i,r,t} = P^{L}_{i,r,t} L_{i,r,t} + P^{Q}_{i,r,t} Q_{i,r,t} + P^{TE}_{i,r,t} TE_{i,r,t} + P^{RN}_{i,r,t} RN_{i,r,t} \quad (6)$$

s.t. (CES option)

$$\left(\frac{VA_{i,r,t}}{PGF_{r,t}}\right)^{1-\frac{1}{\sigma_{VA_i}}} = a^{L}_{i,r} L_{i,r,t}^{1-\frac{1}{\sigma_{VA_i}}} + a^{Q}_{i,r} Q_{i,r,t}^{1-\frac{1}{\sigma_{VA_i}}} + a^{RN}_{i,r} RN_{i,r,t}^{1-\frac{1}{\sigma_{VA_i}}} + a^{TE}_{i,r} TE_{i,r,t}^{1-\frac{1}{\sigma_{VA_i}}} \quad (7)$$

or s.t. (Cobb-Douglas option)

$$VA_{i,r,t} = A_{i,r} PGF_{r,t} {L_{i,r,t}}^{\gamma^L_{i,r}} {Q_{i,r,t}}^{\gamma^Q_{i,r}} {TE_{i,r,t}}^{\gamma^{TE}_{i,r}} {RN_{i,r,t}}^{\gamma^{RN}_{i,r}} \quad (7')$$

and

$$\min P^Q_{i,r,t} Q_{i,r,t} = P^K_{i,r,t} KTOT_{i,r,t} + P^H_{i,r,t} H_{i,r,t} \quad (8)$$

$$\text{s.t.} \quad {Q_{i,r,t}}^{1-\frac{1}{\sigma_{CAP_i}}} = a^K_{i,r} {KTOT_{i,r,t}}^{1-\frac{1}{\sigma_{CAP_i}}} + a^H_{i,r} {H_{i,r,t}}^{1-\frac{1}{\sigma_{CAP_i}}} \quad (9)$$

The capital stock in region s is described by:

$$KTOT_{i,s,t} = \sum_r K_{i,r,s,t} \quad (10)$$

Comment: in this model, production quotas have been introduced. For the associated sectors, production is equal to the quota and an additional income, equal to $P^{Quota}_{i,r,t} Quota_{i,r,t}$, is drawn from the quota.

4.2 Demand

Determination of demand results from the following optimization programs:

LES-CES (first stage)

$$\min P_{r,t} UT_{r,t} = \sum_i P^C_{i,r,t} (C_{i,r,t} - cmin_{i,r}) \quad (11)$$

$$\text{s.t.} \quad {UT_{r,t}}^{1-\frac{1}{\sigma_C}} = \sum_i a^C_{i,r} (C_{i,r,t} - cmin_{i,r})^{1-\frac{1}{\sigma_C}} \quad (12)$$

$$BUDC_{r,t} = \sum_i P^C_{i,r,t} C_{i,r,t} \quad (13)$$

$$P^C_{i,r,t} = P^{DEMTOT}_{i,r,t} (1 + taxcc_{i,r}) \quad (14)$$

$$P^{KG}_{i,r,t} = P^{DEMTOT}_{i,r,t} (1 + taxkgc_{i,r}) \quad (15)$$

$$DEMTOT_{i,r,t} = C_{i,r,t} + \sum_j IC_{i,j,r,t} + KG_{i,r,t} \quad (16)$$

Groups of regions (second stage)

$$\min P^{DEMTOT}_{i,r,t} DEMTOT_{i,r,t} = P^{DEMU}_{i,r,t} DEMU_{i,r,t} + P^{DEMV}_{i,r,t} DEMV_{i,r,t} \quad (17)$$

$$\text{s.t.} \quad {DEMTOT_{i,r,t}}^{1-\frac{1}{\sigma_{GEO_i}}} = a^{DEMU}_{i,r} {DEMU_{i,r,t}}^{1-\frac{1}{\sigma_{GEO_i}}} + a^{DEMV}_{i,r} {DEMV_{i,r,t}}^{1-\frac{1}{\sigma_{GEO_i}}} \quad (18)$$

Armington (third stage)

$$\min P^{DEMU}_{i,r,t} DEMU_{i,r,t} = P^D_{i,r,t} D_{i,r,t} + P^M_{i,r,t} M_{i,r,t} \quad (19)$$

$$\text{s.t.} \quad {DEMU_{i,r,t}}^{1-\frac{1}{\sigma_{ARM_i}}} = a^{DEM}_{i,r} {D_{i,r,t}}^{1-\frac{1}{\sigma_{ARM_i}}} + a^M_{i,r} {M_{i,r,t}}^{1-\frac{1}{\sigma_{ARM_i}}} \quad (20)$$

Regions (fourth stage)
For foreign regions with the same level of development:

$$\min P^{M}_{i,s,t} M_{i,s,t} = \sum_{r \in U(s)} P^{DEM}_{i,r,s,t} DEM_{i,r,s,t} \tag{21}$$

$$\text{s.t.} \quad M^{1-\frac{1}{\sigma_{IMP_i}}}_{i,s,t} = \sum_{r \in U(s)} a^{IMP}_{i,r,s} DEM^{1-\frac{1}{\sigma_{IMP_i}}}_{i,r,s,t} \tag{22}$$

For foreign regions with different levels of development:

$$\min P^{DEMV}_{i,s,t} DEMV_{i,s,t} = \sum_{r \in V(s)} P^{DEM}_{i,r,s,t} DEM_{i,r,s,t} \tag{23}$$

$$\text{s.t.} \quad DEMV^{1-\frac{1}{\sigma_{IMP_i}}}_{i,s,t} = \sum_{r \in V(s)} a^{IMP}_{i,r,s} DEM^{1-\frac{1}{\sigma_{IMP_i}}}_{i,r,s,t} \tag{24}$$

Varieties (fifth stage)

$$DEMVAR_{i,r,s,t} = DEM_{i,r,s,t} NB^{1-\frac{1}{\sigma_{VAR_i}}}_{i,r,t} \tag{25}$$

$$P^{DEM}_{i,r,s,t} = P^{DEMVAR}_{i,r,s,t} NB^{1-\frac{1}{\sigma_{VAR_i}}}_{i,r,t} \tag{26}$$

$$DVAR_{i,s,t} = D_{i,s,t} NB^{1-\frac{1}{\sigma_{VAR_i}}}_{i,s,t} \tag{27}$$

$$P^{D}_{i,s,t} = P^{DVAR}_{i,r,t} NB^{1-\frac{1}{\sigma_{VAR_i}}}_{i,s,t} \tag{28}$$

Intermediate consumption

$$P^{IC}_{i,j,r,t} = P^{DEMTOT}_{i,r,t} (1 + taxicc_{i,j,r}) \tag{29}$$

$$\min P^{CNTER}_{j,r,t} CNTER_{j,r,t} = \sum_{i} P^{IC}_{i,j,r,t} IC_{i,j,r,t} \tag{30}$$

$$\text{s.t.} \quad CNTER^{1-\frac{1}{\sigma_{IC}}}_{j,r,t} = \sum_{i} a^{IC}_{i,j,r} IC^{1-\frac{1}{\sigma_{IC}}}_{i,j,r,t} \tag{31}$$

Capital good

$$\min P^{INVTOT}_{r,t} INVTOT_{r,t} = \sum_{i} P^{KG}_{i,r,t} KG_{i,r,t} \tag{32}$$

$$\text{s.t.} \quad INVTOT^{1-\frac{1}{\sigma_{KG}}}_{r,t} = \sum_{i} a^{KG}_{i,r} KG^{1-\frac{1}{\sigma_{KG}}}_{i,r,t} \tag{33}$$

Commodity market equilibrium

Imperfect competition

$$Y_{i,r,t} = DVAR_{i,r,t} + \sum_s DEMVAR_{i,r,s,t} \tag{34}$$

$$TRADE_{i,r,s,t} = NB_{i,r,t} DEMVAR_{i,r,s,t} \tag{35}$$

Perfect competition

$$Y_{i,r,t} = D_{i,r,t} + \sum_s DEM_{i,r,s,t} \quad (i \notin TrT) \tag{36}$$

$$Y_{i_{TrT},r,t} = D_{i_{TrT},r,t} + \sum_s DEM_{i_{TrT},r,s,t} + TRM_{i_{TrT},r,t} \tag{37}$$

$$TRADE_{i,r,s,t} = DEM_{i,r,s,t} \tag{38}$$

Transport sector

Transport demand

$$TR_{i,r,s,t} = \mu_{i,r,s} TRADE_{i,r,s,t} \tag{39}$$

$$MONDTR_t = \sum_{i,r,s} TR_{i,r,s,t} \tag{40}$$

Transport supply

$$P^Y_{i_{TrT},r,t}(1 + taxp_{i_{TrT},r}) TRM_{i_{TrT},r,t} = \theta_{i_{TrT},r} P^T_t MONDTR_t \tag{41}$$

$$MONDTR_t = a^T \prod_r TRM_{i_{TrT},r,t}{}^{\theta_{i_{TrT},r}} \tag{42}$$

4.3 Factor market

Labour market
Developed countries: labour allocation between agricultural and non agricultural sectors

$$L^{Agri}_{r,t} = b^{L^{Agri}}_r \overline{L}_{r,t} \left(\frac{P^{L^{Agri}}_{r,t}}{P^{\overline{L}}_{r,t}} \right)^{\sigma_L} \tag{43}$$

$$L^{notAgri}_{r,t} = b^{L^{notAgri}}_r \overline{L}_{r,t} \left(\frac{P^{L^{notAgri}}_{r,t}}{P^{\overline{L}}_{r,t}} \right)^{\sigma_L} \tag{44}$$

Developing countries: dual labour market

$$P_{r,t}^{L^{notAgri}} = P_{r,t,Ref}^{L^{notAgri}} \prod_i \left(\frac{P_{i,r,t}^C}{P_{i,r,Ref}^C} \right)^{\frac{P_{i,r,t_0}^C C_{i,r,t_0}}{\sum_j P_{j,r,t_0}^C C_{j,r,t_0}}} \tag{45}$$

$$L_{r,t}^{Agri} = L_{r,t,Ref}^{Agri} \tag{46}$$

where $L_{r,t,Ref}^{notAgri}$ and $L_{r,t,Ref}^{Agri}$ are the baseline (Ref) labour supply exogenously calculated from migration flows in FAO data.
$P_{r,t,Ref}^{L^{notAgri}}$ is computed endogenously from $L_{r,t,Ref}^{notAgri}$ in the baseline.

Labour market (both cases)

$$P_{r,t}^{\overline{L}} \overline{L}_{r,t} = P_{r,t}^{L^{Agri}} L_{r,t}^{Agri} + P_{r,t}^{L^{notAgri}} L_{r,t}^{notAgri} \tag{47}$$

Land market

$$W_{i,r,t}^{TE} = P_{r,t}^{TE} + P_{r,t} \, TsubTE_{i,r,t} \tag{48}$$

Land supply

$$W_{r,t}^{\overline{TE}} \overline{TE}_{r,t} = \sum_i W_{i,r,t}^{TE} TE_{i,r,t} \tag{49}$$

$$\overline{TE}_{r,t} = \overline{TE}_{r,t_0} \left(W_{r,t}^{\overline{TE}} \right)^{\sigma_{\overline{TE}}} \quad (\text{NB} : W_{r,t_0}^{\overline{TE}} = 1) \tag{50}$$

Land allocation

$$TE_{i,r,t} = b_{i,r}^{TE} \overline{TE}_{r,t} \left(\frac{W_{i,r,t}^{TE}}{W_{r,t}^{\overline{TE}}} \right)^{\sigma_{TE}} \tag{51}$$

Full use of factor endowments

$$L_{r,t}^{Agri} = \sum_{j \in Agri(j)} L_{j,r,t} \tag{52}$$

$$L_{r,t}^{notAgri} = \sum_{j \notin Agri(j)} L_{j,r,t} \tag{53}$$

$$\overline{TE}_{r,t} = \sum_j TE_{j,r,t} \tag{54}$$

$$\overline{H}_{r,t} = \sum_j H_{j,r,t} \tag{55}$$

Comments:

- In comparison to the standard model, the agricultural version distinguishes between two types of unskilled labour: agricultural labour and non agricultural labour. A partial mobility between these two types of labour is allowed through a Constant Elasticity of Transformation supply function. Within each category, labour is perfectly mobile.
- A duality of labour has been assumed in developing countries: an efficiency wage scheme determines the level of wages in non agricultural sectors and the corresponding labour demand, while labour demand in agricultural sectors is exogenous. The efficiency wage is set such that the purchasing power of non agricultural wages remains unchanged after the shock.

4.4 Revenues

For imperfectly competitive sectors:

$$\begin{aligned} 0 =& P^Y_{i,r,t}\left(NB_{i,r,t}\sum_s \frac{DEMVAR_{i,r,s,t}}{1+EP_{i,r,s,t}} + \frac{NB_{i,r,t}DVAR_{i,r,t}}{1+EPD_{i,r,t}}\right) \\ & - (P^{VA}_{i,r,t}VA_{i,r,t} + P^{CNTER}_{i,r,t}CNTER_{i,r,t}) \end{aligned} \tag{56}$$

Comment: this corresponds to the zero profit condition allowing to compute the number of firms.

Tax revenue from imperfectly competitive sectors

$$RECPROD_{i,r,t} = taxp_{i,r}P^Y_{i,r,t}\left(NB_{i,r,t}\sum_s \frac{DEMVAR_{i,r,s,t}}{1+EP_{i,r,s,t}} + \frac{NB_{i,r,t}DVAR_{i,r,t}}{1+EPD_{i,r,t}}\right)$$

$$RECEXP_{i,r,t} = (1+taxp_{i,r})P^Y_{i,r,t}NB_{i,r,t} \tag{57}$$

$$* \sum_s (TAXEXP_{i,r,s,t} + taxAMF_{i,r,s,t})\frac{DEMVAR_{i,r,s,t}}{1+EP_{i,r,s,t}} \tag{58}$$

Tax revenue from perfectly competitive sectors

$$RECPROD_{i,r,t} = taxp_{i,r}P^Y_{i,r,t}Y_{i,r,t} \tag{59}$$

$$\begin{aligned} RECEXP_{i,r,t} =& (1+taxp_{i,r})P^Y_{i,r,t} \\ & * \sum_s (TAXEXP_{i,r,s,t} + taxAMF_{i,r,s,t})TRADE_{i,r,s,t} \end{aligned} \tag{60}$$

For both sectors

$$RECDD_{i,r,t} = \sum_{r} DD_{i,r,s,t} P^{CIF}_{i,r,s,t} TRADE_{i,r,s,t} \tag{61}$$

$$RQUOTA_{r,s,t} = \sum_{i \in TQUOTAO} TQUOTA_{i,r,s,t} P^{CIF}_{i,r,s,t} TRADE_{i,r,s,t} \tag{62}$$

$$\begin{aligned} RECCONS_{i,s,t} = {} & P^{DEMTOT}_{i,s,t} (taxcc_{i,s} C_{i,s,t} + taxkgc_{i,s} KG_{i,s,t} \\ & + \sum_{j} taxicc_{i,j,s,t} IC_{i,j,s,t}) \end{aligned} \tag{63}$$

$$\begin{aligned} RECTAX_{r,t} = {} & \sum_{i} RECPROD_{i,r,t} + RECEXP_{i,r,t} \\ & + RECDD_{i,r,t} + RECCONS_{i,r,t} \end{aligned} \tag{64}$$

Savings

$$BUDC_{r,t} = (1 - epa_r) REV_{r,t} \tag{65}$$

Factor mobility

$$P^{L}_{i,r,t} = P^{L^{Agri}}_{r,t} \qquad (i \in Agri(i)) \tag{66}$$

$$P^{L}_{i,r,t} = P^{L^{notAgri}}_{r,t} \qquad (i \notin Agri(i)) \tag{67}$$

$$P^{TE}_{i,r,t} = P^{\overline{TE}}_{r,t} \tag{68}$$

$$P^{H}_{i,r,t} = P^{\overline{H}}_{r,t} \tag{69}$$

4.5 Prices definition

Sale price (imperfect competition)

$$P^{DEMVAR}_{i,r,s,t} = P^{CIF}_{i,r,s,t} (1 + DD_{i,r,s,t}) \tag{70}$$

$$P^{DVAR}_{i,r,t} = \frac{P^{Y}_{i,r,t} (1 + taxp_{i,r})}{1 + EPD_{i,r,t}} \tag{71}$$

CIF price (imperfect competition)

$$P^{CIF}_{i,r,s,t} = (1 + taxp_{i,r})(1 + TAXEXP_{i,r,s,t} + taxAMF_{i,r,s,t}) \frac{P^{Y}_{i,r,t}}{1 + EP_{i,r,s,t}} + \mu_{i,r,s} P^{T}_{t} \tag{72}$$

Sale price (perfect competition)

$$P^{DEM}_{i,r,s,t} = P^{CIF}_{i,r,s,t}(1 + DD_{i,r,s,t}) \tag{73}$$

$$P^{D}_{i,r,t} = P^{Y}_{i,r,t}(1 + taxp_{i,r}) \tag{74}$$

CIF price (perfect competition)

$$P^{CIF}_{i,r,s,t} = (1 + taxp_{i,r})(1 + TAXEXP_{i,r,s,t} + taxAMF_{i,r,s,t})P^{Y}_{i,r,t} + \mu_{i,r,s}P^{T}_{t} \tag{75}$$

4.6 Imperfect competition

Determination of market shares

$$SDU_{i,s,t} = \frac{P^{D}_{i,s,t}D_{i,s,t}}{P^{DEMU}_{i,s,t}DEMU_{i,s,t}} \tag{76}$$

$$SDT_{i,s,t} = \frac{P^{D}_{i,s,t}D_{i,s,t}}{P^{DEMTOT}_{i,s,t}DEMTOT_{i,s,t}} \tag{77}$$

$$SE_{i,r,s,t} = \frac{P^{DEM}_{i,r,s,t}DEM_{i,r,s,t}}{P^{M}_{i,s,t}M_{i,s,t}} \tag{78}$$

$$SU_{i,r,s,t} = \frac{P^{DEM}_{i,r,s,t}DEM_{i,r,s,t}}{P^{DEMU}_{i,s,t}DEMU_{i,s,t}} \tag{79}$$

$$SV_{i,r,s,t} = \frac{P^{DEM}_{i,r,s,t}DEM_{i,r,s,t}}{P^{DEMV}_{i,s,t}DEMV_{i,s,t}} \tag{80}$$

$$Sh_{i,r,s,t} = \frac{P^{DEM}_{i,r,s,t}DEM_{i,r,s,t}}{P^{DEMTOT}_{i,s,t}DEMTOT_{i,s,t}} \tag{81}$$

Mark-up in domestic markets

$$\begin{aligned} NB_{i,r,t}(EPD_{i,r,t} + \frac{1}{\sigma_{VAR_i}}) = &\left[\frac{1}{\sigma_{VAR_i}} - \frac{1}{\sigma_{ARM_i}}\right] + \left[\frac{1}{\sigma_{ARM_i}} - \frac{1}{\sigma_{GEO_i}}\right] SDU_{i,r,t} \\ &+ \left[\frac{1}{\sigma_{GEO_i}} - \frac{1}{\sigma_{C_i}}\right] SDT_{i,r,t} \end{aligned} \tag{82}$$

Mark-up in foreign markets in countries with the same level of development

$$\begin{aligned} NB_{i,r,t}(EP_{i,r,s,t} + \frac{1}{\sigma_{VAR_i}}) = &\left[\frac{1}{\sigma_{VAR_i}} - \frac{1}{\sigma_{ARM_i}}\right] + \left[\frac{1}{\sigma_{IMP_i}} - \frac{1}{\sigma_{ARM_i}}\right] SE_{i,r,s,t} \\ &+ \left[\frac{1}{\sigma_{ARM_i}} - \frac{1}{\sigma_{GEO_i}}\right] SU_{i,r,s,t} + \left[\frac{1}{\sigma_{GEO_i}} - \frac{1}{\sigma_{C_i}}\right] Sh_{i,r,s,t} \end{aligned} \tag{83}$$

Mark-up in foreign markets in countries with different levels of development

$$NB_{i,r,t}(EP_{i,r,s,t} + \frac{1}{\sigma_{VAR_i}}) = \left[\frac{1}{\sigma_{VAR_i}} - \frac{1}{\sigma_{ARM_i}}\right] + \left[\frac{1}{\sigma_{IMP_i}} - \frac{1}{\sigma_{GEO_i}}\right] SV_{i,r,s,t} + \left[\frac{1}{\sigma_{GEO_i}} - \frac{1}{\sigma_{C_i}}\right] Sh_{i,r,s,t} \quad (84)$$

4.7 Intervention price scheme (European Union)

Mode 0: no subsidy change

$$TAXEXP_{i,r,s,t} = TAXEXP_{i,r,s,t_0} \quad (85)$$

Mode 1: no subsidy

$$TAXEXP_{i,r,s,t} = 0 \quad (86)$$

Mode 2: perfect competition

$$P^{Y}_{i,r_{EU},t} = P^{Int}_{i,r,t} \quad (87)$$

Mode 2: imperfect competition

$$\sum_{s} \frac{P^{Y}_{i,r,t}}{1 + EP_{i,r,s,t}} TRADE_{i,r,s,t} = P^{Int}_{i,t} \sum_{s} TRADE_{i,r,s,t} \quad (88)$$

Mode 3: subsidised exports ceiling

$$\sum_{s \neq r} TRADE_{i,r,s,t} = MaxExpSub_{i,r,t} \quad (89)$$

Mode 2 or 3, or subsidy change and subsidy for at least one destination before the change

$$TAXEXP_{i,r,s,t} = TAXREF_{i,r,t}\, TAXEXP_{i,r,s,t_0} \quad (90)$$

Mode 2 or 3, or subsidy change and no subsidy for all destinations before the change

$$TAXEXP_{i,r,s,t} = TAXMOY_{i,r,t} \quad (91)$$

Mode 2 or 3, or subsidy change

$$TAXMOY_{i,r,t} \sum_{s \neq r} TRADE_{i,r,s,t} = \sum_{s \neq r} TAXEXP_{i,r,s,t}\, TRADE_{i,r,s,t} \quad (92)$$

Comments:

The intervention price scheme in the EU is modelled as follows: as soon as the internal price becomes lower than the intervention price, the EU subsidises exports so as to raise the internal price to the level of the intervention price. In actual facts, the EU also increases inventories but inventories are not accounted for MIRAGE.

In practice, the price scheme is divided into 4 possible modes:

- For countries other than the EU or sectors not concerned by intervention prices, the subsidy rate is exogenous.
- When the intervention price is lower than the internal price, there is no export subsidy.
- When the intervention price would be higher than the internal price, the export subsidy rate is endogenous. The distribution across importers is the same as in the baseline. If there was no subsidy in the baseline, this distribution is homogeneous.
- The subsidization of exports is limited by a maximum of subsidized exports from the WTO. If this limit is reached, then this constraint replaces the price constraint.

When a simulation is complete, the model checks if the constraints defining a mode still hold. If they do not, then the mode is changed automatically until there is no more necessary change.

4.8 Investment

$$INV_{i,r,s,t} = a_{i,r,s} B_{r,t} KTOT_{i,s,t} \, \mathrm{e}^{\alpha W^K_{i,s,t}} \tag{93}$$

$$W^K_{i,r,t} = P^K_{i,r,t} + P_{r,t} \, TsubK_{i,r,t} \tag{94}$$

$$INVTOT_{s,t} = \sum_{i,r} INV_{i,r,s,t} \tag{95}$$

4.9 Regional equilibrium

$$GDPVOL_{r,t} * P^{CIndex}_{r,t} = REV_{r,t} + PIBMVAL_t * SOLD_{r,t} \tag{96}$$

$$\text{with} \quad P^{CIndex}_{r,t} = \prod_i \left(\frac{P^C_{i,r,t}}{P^C_{i,r,t_0}} \right)^{\frac{P^C_{i,r,t_0} C_{i,r,t_0}}{\sum_j P^C_{j,r,t_0} C_{j,r,t_0}}} \tag{97}$$

$$\begin{aligned} GDPVOL_{r,t} * P^{CIndex}_{r,t} = & \sum_s (RQUOTA_{r,s,t} - RQUOTA_{s,r,t}) \\ & + RECTAX_{r,t} + \sum_i P^{RN}_{i,r,t} RN_{i,r,t} + \sum_{i,s} (P^K_{i,r,s,t} K_{i,r,s,t}) \\ & + \overline{L}_{r,t} P^{\overline{L}_{r,t}} + \overline{TE}_{r,t} P^{\overline{TE}}_{r,t} + \overline{H}_{r,t} P^{\overline{H}}_{r,t} \end{aligned} \tag{98}$$

$$epa_r REV_{r,t} = \sum_{i,s} P^{INVTOT}_{s,t} INV_{i,r,s,t} \tag{99}$$

$$PIBMVAL_t = \sum_{i,r} PVA_{i,r,t} VA_{i,r,t} \tag{100}$$

4.10 Dynamics

$$K_{i,r,s,t} = K_{i,r,s,t-1}(1-\delta) + INV_{i,r,s,t} \tag{101}$$

$$\overline{L}_{r,t} = \rho_r \overline{L}_{r,t-1} \tag{102}$$

$$\overline{H}_{r,t} = \rho_r \overline{H}_{r,t-1} \tag{103}$$